Decision Tree & Sensitivity Analysis

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A practice exercise to develop a decision tree and then to perform a rollback to determine a preferred choice given a patient's preference for outcome. The data is then taken through a sensitivity analysis.
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Decision Tree Assignment
A construction worker presents with a badly infected compound fracture of the left ankle. The infection is not only threatening to destroy the ankle itself, but is spreading proximally and the septic complications are potentially life threatening. You are faced with the following difficult decision:

(1) Should you perform a below the knee amputation immediately?

(2) Should you perform a surgical debridement followed by antibiotic treatment to save the ankle?

Given outcomes and probabilities

<table>
<thead>
<tr>
<th>Outcomes</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foot saved using antibiotics</td>
<td>0.50</td>
</tr>
<tr>
<td>Full recovery after foot saved</td>
<td>0.80</td>
</tr>
<tr>
<td>Recovery with limp after foot saved using antibiotics</td>
<td>0.20</td>
</tr>
<tr>
<td>Death after infection not controlled by antibiotics</td>
<td>0.10</td>
</tr>
<tr>
<td>Above the knee amputation if infection not controlled by antibiotics</td>
<td>0.80</td>
</tr>
<tr>
<td>Below the knee amputation if infection not controlled by antibiotics</td>
<td>0.10</td>
</tr>
<tr>
<td>Survival after immediate below the knee amputation</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Resulting Decision Tree with trace probability
Without the patients preference considered as to the resulting quality of life they would have (or utility), we can see the traced out probabilities.
Calculated expected value (rollback)
Considering the below preferences provided by the patient about quality of life, we will input for consideration in our decision tree and perform a rollback to calculate expected values on each decision tree node.

Given Utility Values

<table>
<thead>
<tr>
<th>Possible outcome</th>
<th>Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recovery with a limb</td>
<td>0.98</td>
</tr>
<tr>
<td>Recovery with foot amputation</td>
<td>0.70</td>
</tr>
<tr>
<td>Recovery with leg amputation</td>
<td>0.60</td>
</tr>
<tr>
<td>Entire limb saved with no limb</td>
<td>1.00 (assumed)</td>
</tr>
<tr>
<td>Death</td>
<td>0.00 (assumed)</td>
</tr>
</tbody>
</table>

With the above numerical values considered for preferred outcomes, we can see that through comparison the preferred choice, given the expected values, is in trying the surgical debridement followed by antibiotic treatment for the best hoped outcome. .77 (Foot Saved) > .70 (Amputation).

Expected Value Calculations:
Foot Saved = (1.00 x 0.80) + (0.98 x 0.20) = 1
Infection worsens = (0.00 x 0.10) + (0.60 x 0.80) + (0.70 x 0.10) = .55
Surgical debridement & antibiotics = (1 x .50) + (.55 x .50) = .77
Sensitivity Analysis
Using the decision tree you constructed in part 1, try changing the probability of the foot ultimately being saved from .5 to two different values:

Probability of Foot Saved = .2
Comparing the evaluated values we can see below that the preferred choice would be to have an immediate amputation done below the knee.

Probability of Foot Saved = .8
Comparing the evaluated values we can see below that the preferred choice would be to have the surgical procedure done followed by antibiotic treatment to attempt to control the infection with homes toward a full recovery.
Plotted values

The below graph demonstrates the probability threshold where both expected values when considered for “Foot Saved” are plotted on the graph and a line drawn through three values to show a line. The circled point in the middle shows the intersection of the .050 which was already provided. As said in the exercise, this point represents “where the expected value of the two branches is equal”.

The horizontal line on the plot represents the utility given by the patient of .70 which represents amputation below the knee. The point at which the line diagonal line intersects with the utility score of .70 represents a probability threshold of .34. At the point the probability of “Foot Saved” is lower than this threshold immediate amputation would be better. Otherwise, if the probability is equal to or greater than 34%, surgical debridement and antibiotics would be a better choice.

This is a valuable tool because it visually demonstrates the working decision paths where the threshold is set and choices yielding an outcome less than less than the circled point would mean that immediate amputation would be a better one. It can be used in conjunction with sensitivity adjustments to see on the line where the outcome lies.
Data point already calculated for Prob (Foot Saved) = 0.50

Expected Values for Amputation branch
(no change because this variable is not varied)